

نام و نام خانوادگی: شماره دانشجویی: آزمون پایان ترم تحقیق پیشرفته ۸ دی ماه ۹۲ زمان: ۱۲۰ دقیقه

1. Show that one and only one of the following two systems has a solution:

System 1:  $Ax = 0, \quad x \geq 0, \quad cx > 0,$

System 2:  $wA \geq c, \quad w = \text{unrestricted}.$

2. a) Consider  $\min cx$  subject to  $\{Ax \geq 0, x \geq 0\}$ . Show that KKT conditions are necessary and sufficient conditions for optimality.

b) Write the KKT optimality conditions for the following problem:

$\min cx$  subject to  $\{A_1x = b_1, \quad A_2x \geq b_2, \quad x \geq 0\}.$

3. Use the simplex method for bounded variables to solve the following problem:

$$\begin{aligned} \min \quad & 3x_1 - 4x_2, \\ & 3 \leq x_1 + x_2 \leq 4 \\ & -15 \leq 3x_1 - 5x_2 \leq 2 \\ & x_1, x_2 \geq 0. \end{aligned}$$

4. Show that in the revised simplex method, if  $\bar{b}_r < 0$  and  $y_{rj} \geq 0$  for  $j = 1, \dots, n$ , then the dual is unbounded (and the primal is infeasible) by constructing a suitable direction. (Hint: consider  $w = c_B B^{-1} + \lambda B^r$ , where  $B^r$  is the  $r$ th row of  $B^{-1}$ .)

5. Consider the following problem:

$$\begin{aligned} \min \quad & -x_1 + x_2 - 2x_3 \\ & x_1 + x_2 + 2x_3 \leq 6, \\ & -x_1 + 2x_2 + 3x_3 \leq 9 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

a) Suppose that the vector  $c = (-1, 1, -2)$  is replaced by  $(-1, 1, -2) + \lambda(2, -2, 3)$  where  $\lambda$  is a nonnegative number. Find optimal solutions for all values of  $\lambda$ .

b) Using sensitivity analysis, find a new optimal solution if  $c_3$  is changed from  $-2$  to  $3$ .

c). Using sensitivity analysis, find a new optimal solution if  $a_3$  is changed from  $(2, 3)^t$  to  $(-4, 1)^t$ .

d). Suppose that the variable  $x_3$ , is deleted from problem. What happens to the optimal solution.

6. If P (primal problem) is unbounded, is it possible to change its right hand side and make it have a finite optimum?